

Erratum: Josephson junctions and AdS/CFT networks

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In the recent paper [1] we put forward a new holographic framework for the description of Josephson junctions (JJs) and Josephson junction networks (JJNs). A simple illustration of the method describing a particular JJ was presented in section 3 of [1]. In this erratum we would like to clarify some key aspects of the discussion of the Josephson current that appeared in subsection 3.2 of [1], in particular in what sense eq. (3.9) in [1] gives a sensible definition of a Josephson current in the system described in section 3.

In section 3 we considered a system of two sites that represents two infinitely thin layers of superconducting material at zero charge density coupled through a weak link expressed by an interaction which can be suggestively written as

$$\mathcal{W}(\mathcal{O}_1, \mathcal{O}_2) = h \left(e^{i\vartheta} \mathcal{O}_1 \mathcal{O}_2^\dagger + e^{-i\vartheta} \mathcal{O}_1^\dagger \mathcal{O}_2 \right) = \mathcal{W}_E + \mathcal{W}_{J_{ext}} \quad (1)$$

with the definition

$$\mathcal{W}_E = h \cos \vartheta \left(\mathcal{O}_1 \mathcal{O}_2^\dagger + \mathcal{O}_1^\dagger \mathcal{O}_2 \right), \quad \mathcal{W}_{J_{ext}} = ih \sin \vartheta \left(\mathcal{O}_1 \mathcal{O}_2^\dagger - \mathcal{O}_1^\dagger \mathcal{O}_2 \right). \quad (2)$$

\mathcal{W}_E is an interaction that mediates no interlayer charge transfer. In contrast, $\mathcal{W}_{J_{ext}}$ is an interaction based on the charge-transferring operator

$$J_{ext} = i \left(\mathcal{O}_1 \mathcal{O}_2^\dagger - \mathcal{O}_1^\dagger \mathcal{O}_2 \right). \quad (3)$$

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The coupling A of this operator in $\mathcal{W}_{J_{ext}}$, namely

$$\mathcal{W}_{J_{ext}} = A J_{ext}, \quad A = h \sin \vartheta, \quad (4)$$

can be interpreted as a new interlayer background gauge potential component. In this sense, for non-vanishing h, ϑ our two-site system lies in an external transverse gauge field and J_{ext} is an externally imposed current.

The total current running across the sites of the junction can be determined in the standard way from an infinitesimal relative $U(1)$ gauge transformation of the action. For an interaction of the form (1) the infinitesimal transformation

$$\delta \mathcal{O}_1 = i\epsilon \mathcal{O}_1, \quad \delta \mathcal{O}_2 = -i\epsilon \mathcal{O}_2 \quad (5)$$

gives the Lagrangian variation

$$\delta \mathcal{L} = 2ih\epsilon \left(e^{i\vartheta} \mathcal{O}_1 \mathcal{O}_2^\dagger - e^{-i\vartheta} \mathcal{O}_1^\dagger \mathcal{O}_2 \right) = \epsilon (J_{site\ 1} - J_{site\ 2}) = 2\epsilon J_{tot}. \quad (6)$$

The second equality is the discretized version of the gradient ∂J across the interlayer direction. In the third equality we used charge conservation to set $J_{site\ 1} = -J_{site\ 2} = J_{tot}$.

Consequently,

$$J_{tot} = h \left(e^{i\vartheta} \mathcal{O}_1 \mathcal{O}_2^\dagger - e^{-i\vartheta} \mathcal{O}_1^\dagger \mathcal{O}_2 \right). \quad (7)$$

Hence, in the vacuum governed by the algebraic equations (3.3) in [1] one finds (at leading order in the $1/N$ expansion)

$$J_{tot} = h \left(e^{i\vartheta} \alpha_1 \alpha_2^* - e^{-i\vartheta} \alpha_1^* \alpha_2 \right) = 0. \quad (8)$$

This is precisely what one expects. Since our system is kept at zero charge density, in the equilibrium state charge cannot flow across the junction between the two sites. As a result, irrespective of the initial configuration, once the interaction (1) is turned on the system backreacts and evolves to a new vacuum, which can be conveniently determined with holographic techniques. In the new vacuum, which is characterized by the solutions of the algebraic equations (3.3) in [1], the condensate phase difference is $\vartheta_{12} = -\vartheta$ and the magnitude of the condensate has adapted accordingly and in direct relation to the strength of the interlayer couplings h, ϑ .

It is interesting to consider the VEV of the externally forced current operator J_{ext} in the new vacuum. At leading order in the $1/N$ expansion

$$\langle J_{ext} \rangle = i(\alpha_1^* \alpha_2 - \alpha_1 \alpha_2^*). \quad (9)$$

Using eq. (3.6) in [1] we obtain

$$\langle J_{ext} \rangle = I_{\max} \sin \vartheta = -I_{\max} \sin \vartheta_{12} \quad (10)$$

with I_{\max} as described in eqs. (3.10)-(3.12) of [1].

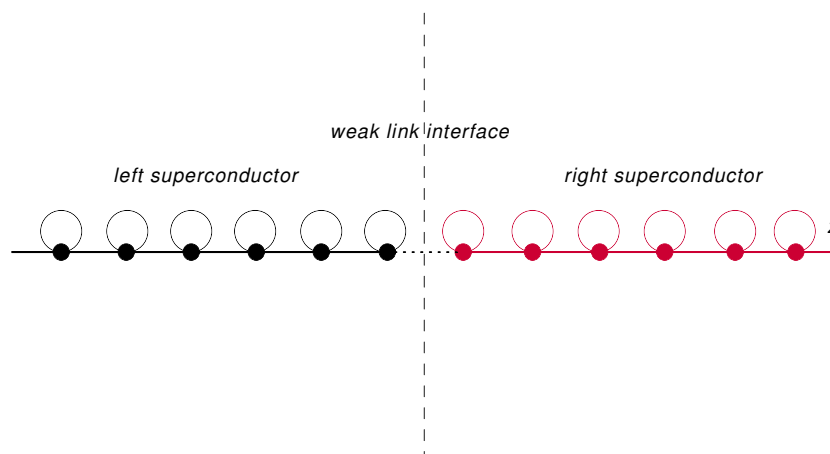


Figure 1. An (un)conventional JJ constructed as two semi-infinite AdS/CFT arrays of (2+1)-dimensional holographic superconductors linked to each other at a two-dimensional weak-link interface.

The vanishing of the total interlayer current J_{tot} implies that the backreaction has created an equal and opposing ‘Josephson current’ $J_{\text{Josephson}}$ (due to the condensate phase difference), which cancels the contribution of the externally imposed $\langle J_{\text{ext}} \rangle$. Specifically,

$$J_{\text{Josephson}} = -\langle J_{\text{ext}} \rangle = I_{\text{max}} \sin \vartheta_{12} \quad (11)$$

in agreement with the expected sine law described in eq. (3.8) of [1].

It is clear that the system we have just described is a peculiar Josephson junction unlike the typical Josephson junction commonly discussed in the literature and engineered in the laboratory. In contrast to our system in a typical junction the superconductor components have finite spatial thickness in the transverse junction direction, charge can flow in this direction, the condensate phase difference is dialed by choice (and not determined dynamically) and a Josephson current arises without having to apply a gauge field across the weak link. Such more conventional junctions can be engineered in our framework in the following way.

Assume we want to describe a junction of two superconductors in three spatial dimensions linked weakly across the third direction z at a two-dimensional interface. Ref. [1] explains how to deconstruct (3 + 1)-dimensional layered superconductors from an array of (2 + 1)-dimensional holographic superconductors using linear AdS/CFT arrays. To engineer an (un)conventional Josephson junction of two superconductors of this type a possible strategy is described in figure 1. A layered superconductor on the left (right) is deconstructed as an array of sites linked through interactions of the form

$$\sum_n h_{L(R)} \left(\mathcal{O}_n^{L(R)} \mathcal{O}_{n+1}^{L(R)\dagger} + \mathcal{O}_n^{L(R)\dagger} \mathcal{O}_{n+1}^{L(R)} \right) . \quad (12)$$

With a real coupling $h_{L(R)}$ no external transverse gauge field is applied along the z direction. Across the two-dimensional interface the right-most black site of the left chain can be linked to the left-most red site of the right chain through a link of a different type depending on

the specific nature of the left and right sites. For s -wave holographic superconductors this could again be of the type described in (12). Then one can solve the analog of the equations (4.3) of [1] and determine the total current across the junction as described above. SNS-type solutions of a uniform array with $h_L = h_R = h_{link}$ were described in subsections 4.1, 4.2, 4.3 of [1] (in a discrete or continuum limit). In general, the asymptotic difference of the phase of the condensates, $\Delta\vartheta = \vartheta_L - \vartheta_R$, is a dialed quantity in these systems. For conventional SNS or SIS-type JJs we anticipate the presence of a Josephson current that follows the sine law relation $I_{\max} \sin \Delta\vartheta$. We hope to return to a detailed survey of such systems in future work.

The above clarifications affect the following points in the remaining discussion of ref. [1]. In subsection 3.3 the Josephson current quoted below eq. (3.13) refers now more correctly to eq. (11). Finally, the discussion about the Josephson current across a chain in subsection 4.4 should be adapted in the obvious manner. The current $I_{n-1,n}$ in [1] refers more appropriately to the VEV $\langle J_{n-1,n} \rangle$ of the externally imposed current defined as in eq. (3) above. This quantity is in general site-dependent. The total current across the chain is determined by the conserved current in eq. (4.13), which is site-independent. For an infinite or periodic chain this current is in general non-vanishing.

References

- [1] E. Kiritsis and V. Niarchos, *Josephson Junctions and AdS/CFT Networks*, *JHEP* **07** (2011) 112 [[arXiv:1105.6100](#)] [[INSPIRE](#)].